11. 325 studs A total of 1500 middle school students were surveyed. The results showed that $\frac{1}{4}$ of the students have a pet living in their homes, $\frac{1}{5}$ have a grandparent living in their homes and $\frac{1}{3}$ have a baby living in their homes. What is the least possible number of students who have no pets, grandparents or babies living in their homes?

$\frac{1}{4}(1500) = 375$
$\frac{1}{5}(1500) = 300$
$\frac{1}{3}(1500) = 500$
1175

1500
−1175
325

12. 3 The first term in a sequence is 5, and each subsequent term in the sequence is the units digit of 2 more than the square of the preceding term. What is the 100th term in the sequence?

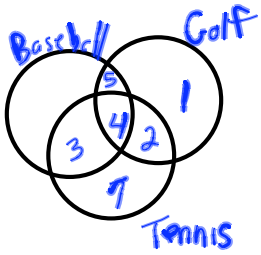
5, 7, 1, 3, 1, 3 ....

5^2+2 7^2+2 1^2+2 3^2+2

1st 2 terms 5, 7 remaining terms 100−2 = 98
alternate 1, 3, 1, 3 ... since last term is even the 3

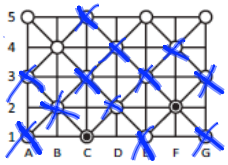
213. ____ studs

Seventy-eight students participate in one or more of three sports: baseball, tennis and golf. Four students participate in all three sports. Five students play only baseball and golf. Two students play only tennis and golf. Three students play only baseball and tennis. If seven students play only tennis, and one student plays only golf, what is the total number of students who play only baseball?



Add #s = 22
78 − 22 = 56

214. B4, E5 The lines in the grid shown indicate "sight lines." A soldier standing at one location can see a soldier standing at another location only if the soldiers are on the same sight line. Two soldiers are located at C1 and F2, respectively. What are the locations of two other soldiers if none of the four soldiers are able to see any of the others?

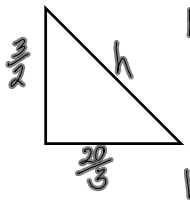


Cross out all positions in soldiers C1, F2 line of sight. } leaves {A5, B4, E5

Since A5 is in B4 & E5's sight line can't use it

So B4, E5

215. _____ A right triangle has legs of length $\frac{3}{2}$ units and $\frac{20}{3}$ units. If the triangle has a perimeter of x units and an area of y units^2, what is the value of $x^2 - y^2$?



$h^2 = (3/2)^2 + (20/3)^2$

$9/4 + 400/9$

$81/36 + 1600/36$

$h = \sqrt{1681/36} = 41/6$

$P = 3/2 + 20/3 + 41/6$

$= 9/6 + 40/6 + 41/6$

$= 90/6 = 15$

$A = \frac{1}{2}(3/2)(20/3) = 5$

$x^2 - y^2$

$15^2 - 5^2 = 225 - 25 = 200$

216. _____ What is the coefficient of $x^2y^2z^2$ in the expansion of $(x + y + z)^6$?

Need coefficient of $x^2y^2z^2$ term

$(x+y+z)^6 = (x+y+z)(x+y+z)(x+y+z)(x+y+z)(x+y+z)(x+y+z)$

So how many ways can we choose 2xs, 2ys + 2zs from 6 sets of ( )s

So ${}_6C_2 = xs = \frac{6!}{4!2!} = \frac{6 \cdot 5}{2 \cdot 1} = 15$

${}_4C_2 = ys = \frac{4!}{2!2!} = \frac{4 \cdot 3}{2 \cdot 1} = 6$

(left) ${}_2C_2 = zs = \frac{2!}{2!1!} = 1$

so $15 \cdot 6 \cdot 1 = 90$ ways

217. _____ A two-digit positive integer is randomly selected. What is the probability that the units digit is a multiple of the tens digit? Express your answer as a common fraction.

90 2-digit integers from 10-99

zero = multiple of any whole #

so 10 integers in teens

5 integers in twenties

4 multiples of 3 in thirties

3 " " 4 in 40s

2 " " 5 in 50s

2 " " 6 " 60s

2 " " 7 " 70s = 32

2 " " 8 " 80s

2 " " 9 " 90s

_____ coins In Quaternion, the coin with the least value is the qua. Four quas equal one quab, four quabs equal one quac, four quacs equal one quad, four quads equal one quae and four quaes equal one quaf. What is the least number of coins that have a combined value of 2012 quas?

base 4 system

qua = a

quab = 4a

quac = 16a

quad = 64a

quae = 256a

quaf = 1024a

$2012 = 1024 + 3(256) + 3(64) + 16 + 3(4)$

$= 1$ quaf $+ 3$ quae $+ 3$ quad, 1 quac $+ 3$ quabs

$= 11$ coins

19. _____ prs How many different pairs of numbers (m, n) can be formed using numbers from the list of integers {1, 2, 3, ... , 99, 100} such that $m < n$ and $m + n$ is even?

m & n must both be even or both odd.

(2,4), (2,6), (4,6), (2,8), (4,8), (6,8), ⋮

49 → pairs (2,100) (4,100) (6,100), ..., (98,100)

$49 \cdot 50 \div 2 = 1225$ even pairs & 1225 odd pairs also

So (2450) pairs

20. _____ units  In the figure shown, right angles are as marked, AG = 4 units and FG = FE. If FG:DE:CB = 1:2:3, what is the length of $\overline{CE}$?

$\triangle AFG$ & $\triangle ADE$, $\triangle ACB$ are similar since each have 90° ∠ & share ∠A

Ratios of corresponding sides are equal.

$AG:GF = AE:ED = AB:BC$ so $\frac{4}{x} = \frac{AE}{2x} = \frac{AB}{3x}$

$AE = 8$, $AB = 12$

DEFG is a kite, so $DG = DE = 2x$

$AD = 4 + 2x$

$AD = \sqrt{8^2 + (2x)^2} = \sqrt{64 + 4x^2}$

$(4 + 2x)^2 = \left(\sqrt{64 + 4x^2}\right)^2$

$16 + 16x + 4x^2 = 64 + 4x^2$

$16 + 16x = 64$

$16x = 48$

$x = 3$ so $\triangle AFC = 3\text{-}4\text{-}5$ rt $\triangle$

$BC = 9$. Since $AB = 12$ $\triangle ACB = 9\text{-}12\text{-}15$ rt $\triangle$

Because $CA = 15$, $EF = 3$ & $AF = 5$

$CE = 15 - 3 - 5 = (7)$

221. _____ ints How many positive two-digit integers each have the property that the integer's value increases by 75% when its digits are reversed?

2 digit # has value $10A + B$

digits reversed BA with value $10B + A$

BA is 75% > AB so

$10B + A = 1.75(10A + B)$

$10B + A = 7/4\,(10A + B)$

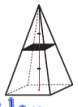
$40B + 4A = 70A + 7B$

$33B = 66A$

$B = 2A$

any # with units digit = twice tens digit will work So 12, 24, 36, & 48 = (4) possible #s

222. _____ cm^3 The square pyramid shown has a volume of 64 cm^3. A plane parallel to the base of the pyramid bisects its altitude and divides the pyramid into two sections, as shown. What is the volume of the smaller pyramid?



altitude of smaller pyramid = ½ alt. of larger & dimensions of its base = ½ dim. of larger base

Small pyramid $V = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$ vol. of larger

So $\frac{1}{8}(64) = (8)$

223. _____ If $x+\frac{4}{x}=y+\frac{4}{y}$ and $x \neq y$, then what is the value of the product xy?

$x+\frac{4}{x}=\frac{x^2+4}{x}$; $y+\frac{4}{y}=\frac{y^2+4}{y}$

$\frac{x^2+4}{x}=\frac{y^2+4}{y}$

$y(x^2+4)=x(y^2+4)$

$x^2y+4y=xy^2+4x$

$x^2y-xy^2=4x-4y$

$xy(x-y)=4(x-y)$

$xy=4$

224. _____ % 

A large square is divided into four congruent squares. Then those four squares are each divided into four smaller congruent squares, some of which are shaded, as shown. What is the probability that a dart thrown at random that lands in the largest square will also land in a shaded region? Express your answer as a percent to the nearest tenth.

2 shaded regions are 3/4 of 1/4 of largest square

Total shaded region $= 2\cdot\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)=\frac{3}{8}$ of area of largest square

$P=\frac{3}{8}=37.5\%$

225. _____ dance teams A 4-person dance team composed of 2 boys and 2 girls is to be selected from a group of 8 girls and 11 boys. How many different dance teams are possible?

${}_8C_2$ ways to choose 2 girls $\frac{8!}{6!2!}=\frac{8\cdot7}{2\cdot1}=28$

${}_{11}C_2=\frac{11\cdot10}{2\cdot1}=55$

Each of 28 pairs of girls can be matched with each of 55 pairs of boys so there are $28\times55=1540$ dance teams.

226. _____ points Mrs. Garcia allowed each student in her class to drop the lowest of their five test scores. When Matt dropped the lowest of his test scores, a 60, his test average increased by 5 points. What is Matt's new test average?

Let Matt's average for 5 tests $=x$.

New average $=\frac{(5x-60)}{4}$

This is 5 more than the old average or $x+5$

So $\frac{5x-60}{4}=x+5$

$5x-60=4x+20$

$x=80=$ old average

$x+5=85=$ new average

227. _____ blocks What is the maximum number of 3" × 1" × 1" blocks that will fit into a box with interior dimensions of 5" × 5" × 10"?

Vol. of smaller block = $3 \cdot 1 \cdot 1 = 3 in^3$

Vol of interior of larger box = $5 \cdot 5 \cdot 10 = 250 in^3$

50 blocks fit horizontally in 10 stack (5 high)

20 " " vertically in 2 columns of 10

12 blocks fit horizontally (rotated 90° in 6 stacks 2 high)

So $2" \times 2" \times 1"$ space unfilled.

$56 + 20 + 12 = (82)$

228. _____ ° Using five identical index cards, the following structure can be created on a flat, level surface. What is the sum of the two marked angles on the side view of this construction?

3 cards in center form equilateral Δ.

∠s = 60°

Since structure rests on flat, level surface, the ∠s on either side of 60° ∠ in middle must add up to $180 - 60 = 120°$.

Let one ∠ = a + the other = b

$a + b = 120$

Since all cards have same length, the triangles on the left + right are isosceles. So another ∠ of a° in left Δ + another ∠ of b° in rt Δ.

Sum of ∠s of Δ = 180°, so

$2a + x + 2b + y = 2(180)$

$2(a+b) + x + y = 360$

$2(120) + x + y = 360$

$240 + x + y = 360$

$x + y = (120)$

_____ mi/h A cruise ship must average 22 mi/h for 10 hours to make its next port on schedule. During the first 4 hours, bad weather caused it to reduce its speed to 16 mi/h. What should its speed be for the remainder of the trip to make it to the next port on schedule?

ship must go $22 \times 10 = 220$ mi between ports.

In 1st 4 hrs it went $4 \cdot 16 = 64$ mi;

so it still has to go $220 - 64 = 156$ mi

To go that far in 6 hr = $156/6 = (26 mi/hr)$

230. _____  Ms. Robinson gives the following homework assignment: You must write an essay by tomorrow. You may work alone or in boy-girl pairs. As it turned out, $\frac{2}{3}$ of the boys and $\frac{3}{5}$ of the girls worked in pairs. What portion of the class worked alone? Express your answer as a common fraction.

2/3 of boys must be same as 3/5 of girls since paired up.

Find common numerator $\frac{6}{9} = \frac{6}{10}$

Since 6 out of 9 parts of the boys were paired with 6 out of 10 parts of the girls

total population = 9 + 10 = 19 parts

So 12/19 students chose to work with a partner, leaving (7/19) of class to work alone.