



61. _____ cubes It takes 6 cubes to build a staircase containing 3 rows. How many cubes are needed to build a staircase that contains 11 rows?



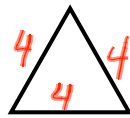
Pattern $3 + 2 + 1 = 6$ for 3 rows
 $4 + 3 + 2 + 1 = 10$ for 4 rows
 \vdots

$11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 66$ cubes
 11 rows

62. _____ cm²



Three congruent, coplanar circles overlap so that each center lies on the other two circles. The diameter of each circle is 8 cm. What is the area of the triangle formed by connecting the centers of the circles? Express your answer in simplest radical form.



Can use Pythagorean theorem or formula for A of Equilateral Δ

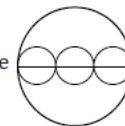
$$A = \frac{s^2 \sqrt{3}}{4} = \frac{4^2 \sqrt{3}}{4} = \frac{16\sqrt{3}}{4} = 4\sqrt{3}$$

63. _____ What is the next number in the geometric sequence: $-1, 4, -16, 64, \dots$?

$x-4$

$$64 \times -4 = -256$$

64. _____ units The centers of three congruent small circles are collinear, and their diameters form the diameter of the large circle, shown, whose area is 81π units². What is the circumference of one of the smaller circles? Express your answer in terms of π .



$$A = 81\pi$$

$$= \pi r^2$$

$$r^2 = 81 \quad r = \sqrt{81} = r = 9$$

$d = 18$
 $18 \div 3$ diameters
 $6 =$ diameter of smaller \circ

$$C = \pi d$$

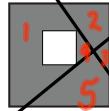
$$= 6\pi$$

65. _____ If $2x + 3 = 4$, what is the value of $12x + 18$?

$$12x + 18 = 6(2x + 3 = 4)$$

$$12x + 18 = 24$$

66. _____ regions



In the figure shown, what is the greatest number of nonoverlapping regions into which the shaded region can be divided with exactly two lines?

5

67. _____ years

Sal collected data on all her family members who were born in the last half of the 20th century. Their birth years are shown in the stem-and-leaf plot. What is the positive difference between the median and the mode of these data?

195	4 4 7 9	
196	0 2 3 3 4 4 5 5 5 6 7 8 8 9	4
197	0 4 4 4 4 8 9	16
198	1 6	8
199	2 4 5 7	2

Key: 195|4 = 1954

Median = middle 17, 18th
= 1968 date

mode - occurs most often = 1974
1974 - 1968 = 6

4
16
8
2
4
34
dates

68. _____ minutes

In a game that lasts 48 minutes, exactly 6 players from each team are on the field at all times. Throughout the game, players are substituted so that 8 players on a team each play an equal amount of time. How many minutes is each of the 8 players on the field during the game?

6 min
8 | 48
6 6 6 6 6 6
P1 P2 P3 P4 P5 P6
at 6+6+6+6+6+6 = 36 P7 P8

69. _____ leaps

A dog is chasing a rabbit that has a head start of 150 ft. If their leaps are synchronized, and the dog leaps 9 ft every time the rabbit leaps 7 ft, in how many leaps will the dog catch up to the rabbit?

x = # of leaps



$$\begin{aligned}
 9x &= 7x + 150 \\
 -7x & \quad -7x \\
 \hline
 2x &= 150 \\
 x &= 75 \text{ leaps}
 \end{aligned}$$

70. _____

For a certain set of five numbers, the mean of all but the largest number is 80, and the mean of all but the smallest number is 90. What is the range of the set of five numbers?

4(80) = 320 (largest missing, smallest included)


4(90) = 360 (smallest missing, largest included)

3 #s in middle in both

so 360 - 320 = 40 difference between largest & smallest

71. _____ teams How many different three-member teams can be formed from a group of six students?

Combination Since order does not matter
 So 6 choose 3 ${}^6C_3 = \frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{3 \cdot 2 \cdot 1 \cdot \cancel{3 \cdot 2 \cdot 1}} = 20$

72. _____ cm²  A right circular cylinder has a volume of 144π cm³ and a height of 9 cm. What is the area of its base? Express your answer in terms of π .

$V = Bh$ $B = \text{base area of figure}$
 $\frac{144\pi}{9} = \frac{B(9)}{9}$
 $16\pi = B$

73. _____ The math team ordered 7 eight-slice pizzas. What fraction of the total amount of the pizza ordered is left after 41 slices are eaten? Express your answer as a common fraction.

$7 \cdot 8 = 56$ slices
 $- 41$ slices eaten
 $\hline 15$ left
 $\frac{15}{56}$

74. _____ The square root of the quantity 3 less than twice a number is equal to 3. What is the number?

$(\sqrt{2x-3})^2 = (3)^2$
 $2x-3 = 9$
 $2x = 12$
 $x = 6$

75. _____ marbles Bailey said to Kaylee, "If you gave me two of your marbles, I'd have twice as many as you'd have." And Kaylee responded, "If you gave me three of your marbles, I'd have three times as many as you'd have." What is the difference between the number of marbles that Bailey and Kaylee have?

$x = \text{Kaylee's marbles}$ $y = \text{Bailey's marbles}$
 $y + 2 = 2(x - 2)$ $y + 3 = 3(x - 3)$
 $y + 2 = 2x - 4$ $2x - 6 + 3 = 3x - 9$
 $y = 2x - 6$ $2x - 3 = 3x - 9$
 $= 2(6) - 6$ $\begin{array}{r} -2x \quad -2x \\ -3 = x - 9 \\ +9 \quad +9 \\ \hline 6 = x \end{array}$
 $= 12 - 6$
 $= 6$
 $6 - 6 = 0$

76. _____ ft If all the angles in the figure shown are right angles, what is the perimeter of the figure?

10
12
10
6
38
(given lengths)

$4x = 12 \text{ ft}$
 $2y = 10 - 3 = 7 \text{ ft}$
 $3z = 10 - 3 = 7 \text{ ft}$

+ 26 = 64

77. _____ strokes

The average of Martha's first 5 rounds of golf is 98 strokes. How many strokes would Martha need to average on her next 3 rounds to bring her average down to 92 strokes?

$5 \cdot 98 = 490$
 $8 \cdot 92 = 736$

$\frac{736 - 490}{3} = 82$

78. _____ On the number line below, the tick marks are evenly spaced. What is the value of $b - a$? Express your answer as a mixed number.

20 tick marks $-8 - (-3) = -8 + 3 = -5$ units

$\frac{5}{20} = \frac{1}{4} = 1 \text{ tick mark}$

$-8 + 5(\frac{1}{4}) = -8 + \frac{5}{4} = -6\frac{3}{4} = b$

$-6\frac{3}{4} + 10(\frac{1}{4}) = -6\frac{3}{4} + 2\frac{1}{2} = -4\frac{1}{4} = a$

Difference $-6\frac{3}{4} + (+4\frac{1}{4}) = -2\frac{2}{4} = -2\frac{1}{2}$

79. _____ dollars The price of a coat that originally sold for \$80 is reduced by 20%. A different coat that originally sold for \$100 is marked down 30%, and then a 10% discount is given on the reduced price. After all of the reductions, what is the positive difference in the prices of the two coats?

$\$80 \cdot (.80) = \64 (20% off - pay 80%)

$\$100 \cdot (.70) = 70$ (30% off - pay 70%)

$\$70 \cdot (.90) = \63 (10% off - pay 90%)

$64 - 63 = \$1$

80. _____ units²

A region in the coordinate plane is bounded by $y = x$, $x = 5$ and $y = 1$. What is the area of this region?

$(1, 1)$, $(5, 1)$, $(5, 5)$

$5 - 1 = 4$

$5 - 1 = 4$

$A = \frac{1}{2} b h$
 $= \frac{1}{2} (4)(4)$
 $= 8$

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