

151. On the number grid shown, Mara colored all of the positive multiples of  $n$ . Once completed, there was exactly one colored square in each column. What is the sum of all possible values of  $n$ ?

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35

1, 2, 3, 4, 5 have more than 1 multiple per column  
 6 and 7 have exactly 1 in each column  
 $6 + 7 = 13$

152. Two standard, six-sided dice are rolled. What is the probability that the positive difference between the numbers rolled is 1? Express your answer as a common fraction.



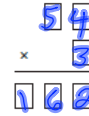
Possibilities  
 5, 6   6, 5   1, 2   2, 1  
 4, 5   5, 4  
 3, 4   4, 3  
 3, 2   2, 3

$$\frac{10}{6 \cdot 6} = \frac{10}{36} = \frac{5}{18}$$

153. What is the slope of a line perpendicular to the segment AB, which has endpoints A(-8.1, 4.9) and B(-7.6, 2.9)? Express your answer as a common fraction.

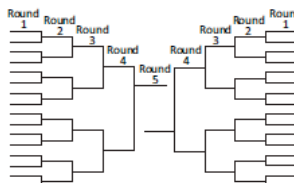
If a line is perpendicular to a segment its slope is the negative reciprocal of slope of segment  
 slope of AB =  $\frac{2.9 - 4.9}{-7.6 - (-8.1)} = \frac{-2}{-7.6 + 8.1} = \frac{-2}{0.5} = \frac{-2}{1/2} = -2 \cdot 2 = -4$   
 slope of line  $\perp$  is  $\frac{1}{4}$

154. Each of the digits 1 to 6 is placed in one of the boxes shown here to correctly complete the multiplication problem. What is the three-digit product?



5 can't be a units digit -  $5 \times \text{even} = 0$   
 and  $5 \times \text{odd} = 5$   
 (don't have 0 or only one 5)  
 product  $< 500$  so 5 is in tens place  
 by trial + error

162

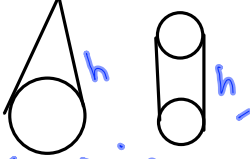
155.  A jousting tournament has 32 competitors in a single elimination bracket, shown here. The table shows the number of points awarded for each correctly predicted match outcome in the tournament. What is the maximum number of points that can be earned?

Round 1	1 pt
Round 2	2 pts
Round 3	4 pts
Round 4	8 pts
Round 5	16 pts

16 Round 1 =  $16 \cdot 1 = 16 \text{ pts}$   
 8 Round 2 =  $8 \cdot 2 = 16 \text{ pts}$   
 4 Round 3 =  $4 \cdot 4 = 16 \text{ pts}$   
 2 Round 4 =  $2 \cdot 8 = 16 \text{ pts}$   
 1 Round 5 =  $1 \cdot 16 = 16 \text{ pts}$

$16 \times 5 = 80 \text{ pts}$

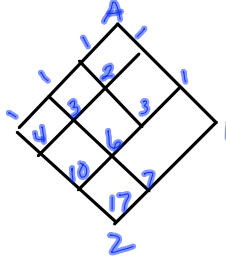
156. \_\_\_\_\_ The circumference of the base of a cone is triple the circumference of a cylinder with the same height. What is the ratio of the volume of the cylinder to the volume of the cone? Express your answer as a common fraction.



$Vol\ of\ cylinder = \pi r^2 h$   
 $Vol\ of\ cone = \frac{1}{3} \pi (3r)^2 h$   
 $= \frac{1}{3} \pi (9r^2) h$   
 $ratio = \frac{\pi r^2 h}{3\pi r^2 h} = \frac{1}{3}$

So cone's radius is 3 times cylinder's

157. \_\_\_\_\_ paths How many paths from A to Z can be traced following line segments on this drawing if paths must be traced in a downward direction, with no retracing?



add # from higher intersections to get # for a new intersection.

(17)

158. \_\_\_\_\_ If  $\frac{x-y}{z-y} = -2$ , what is the value of  $\frac{x-z}{y-z}$ ?

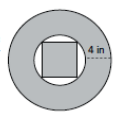
$$x-y = -2z + 2y$$

$$x-y+y = 2y-2z+y$$

$$x = 3y-2z$$

$$\frac{x-z}{y-z} = \frac{3y-2z-z}{y-z} = \frac{3y-3z}{y-z} = \frac{3(y-z)}{y-z} = 3$$

159. \_\_\_\_\_ in<sup>2</sup> In the figure, the square is inscribed in the smaller circle, which has a radius of 4 in. The radius of the larger circle is 8 in. What is the total area of the shaded regions? Express your answer in terms of  $\pi$ .



Area of lg O - smaller O + Area of  $\square$

$$\pi(8)^2 - \pi(4)^2 + (4\sqrt{2})^2$$

$$64\pi - 16\pi + 16\sqrt{4}$$

(48π + 32)

$4^2 + 4^2 = c^2$   
 $32 = c^2$   
 $\sqrt{32} = c$   
 $4\sqrt{2}$

160. \_\_\_\_\_ players In a tennis tournament, each of the 10 competitors plays each other player once. What is the maximum number of players who could end the tournament with a record of 7 or more wins?

Each of 10 competitors plays 9 other players  
 so  $10 \cdot 9 = 90 \div 2 = 45$  matches  
 45 wins + 45 losses  
 if 6 people have 7 wins that's 42 wins  
 remaining 4 can't account for 42 losses  
 since  $4 \cdot 9 = 36 \neq 42$  against each other

If 5 people have 7 wins that's 35 wins  
 remaining 5 have to account for 35 losses  
 $5 \cdot 9 = 45$  so okay

so (5)



Math 101

161. \_\_\_\_\_ ft A rectangular swimming pool, shown here, is surrounded by a concrete deck that is 5 ft wide. The length of the pool is 1.5 times its width, and its area is 216 ft<sup>2</sup>. What is the outside perimeter of the deck?



$$w(1.5w) = 216$$

$$\frac{1.5w^2}{1.5} = \frac{216}{1.5}$$

$$w^2 = 144$$

$$w = 12$$

So  $w+10 = 12+10 = 22$

$$1.5w+10 = \frac{3}{2}(12)+10 = 28$$

□ 22 28 = 30 sq ft = 100

162. \_\_\_\_\_ pm A subway arrives at the station every 25 minutes. A train arrives at the station every 45 minutes. If the subway and the train each arrive at noon, at what time will they next arrive at the station together?

$$25x = 45x$$

$$15x = 3^2 \cdot 5$$

$$3^2 \cdot 5^2$$

3:45 PM

$$9 \cdot 25 = 225 \text{ min}$$

$$\frac{225}{60} = 3 \text{ hr} + 45 \text{ min}$$

$$60 \overline{)225}$$

$$\underline{180}$$

$$45$$

163. \_\_\_\_\_ hours Working together, Tom and Dick can dig 3 holes in 6 hours. Knowing Tom digs twice as fast as Dick, how many hours would it take Tom, working alone, to dig 12 holes?



Since Tom Digs twice as fast as Dick, in 6 hrs, Tom could dig 2 of the holes and Dick would dig 1 hole.

So Tom's rate is 1 hole every 3 hrs

so 12 holes would take  $3 \cdot 12 = 36 \text{ hrs.}$

164. \_\_\_\_\_ times The pages in a book are numbered from 1 to 363. How many times does the digit 3 appear as part of a page number of this book?

From 300 to 363 there are 64 with digit 3 in hundreds

From 30 to 39 10

130 to 139 10

230 to 239 10

330 to 339 10

10 each with 3 in tens place

40

From 3 to 363 there are 37 numbers with 3 in the ones place

$64 + 37 + 40 = 141$

165. \_\_\_\_\_ squares How many squares can be formed on a 16-pin rectangular geoboard?



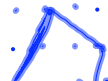
1



2



2



1



9



3



2



1

20

166. In a sequence of five positive integers, each term after the first term is determined by multiplying the preceding term by 1.5. If the median of the five terms is 36, what is the mean of the five terms? Express your answer as a decimal to the nearest tenth.

$$\begin{array}{cccccc}
 16 & 24 & 36 & 54 & 81 \\
 a_1 & a_2 & a_3 & a_4 & a_5 \\
 \frac{3}{2}a_2 = 36 & & & & \\
 a_2 = \frac{2}{3}(36) & & & & \frac{3}{2}(\frac{36}{1.5}) = 54 \\
 = 24 & & & & \\
 \frac{3}{2}a_1 = 24 \text{ so } a_1 = \frac{2}{3}(24) = 16 & & & & \frac{3}{2}(\frac{24}{1.5}) \\
 \frac{16+24+36+54+81}{5} = 42.2
 \end{array}$$

167. The sum of Madison's age and 3 times Harper's age is 47 years. In 2 years Madison will be twice as old as Harper. How old is Harper?

$$\begin{array}{l}
 x = \text{Madison's age} \quad y = \text{Harper's age} \\
 x + 3y = 47 \quad x + 2 = 2(y + 2) \\
 x = 47 - 3y \quad x + 2 = 2y + 4 \\
 47 - 3y + 2 = 2y + 4 \\
 49 - 3y + 2 = 2y + 4 \\
 49 = 5y + 4 \\
 45 = 5y \\
 9 = y
 \end{array}$$

168. Andie bought 3 oldies CDs and 2 current CDs for \$78. Deanne bought 2 oldies CDs and 3 current CDs for \$82. What is the positive difference in the price of an oldie CD and the price of a current CD?

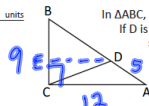


$$\begin{array}{l}
 x = \text{oldie} \quad y = \text{current} \\
 -2(3x + 2y = 78) \quad 2x + 3y = 82 \\
 3(2x + 3y = 82) \quad \phantom{2x + 3y = 82} \\
 \hline
 -6x - 4y = -156 \quad 2x + 3y = 82 \\
 6x + 9y = 246 \quad \phantom{2x + 3y = 82} \\
 \hline
 5y = 90 \\
 y = 18 \\
 2x + 3(18) = 82 \\
 2x + 54 = 82 \\
 2x = 28 \\
 x = 14 \\
 18 - 14 = 4
 \end{array}$$

169. For how many positive integers containing no digit of zero is the sum of the digits equal to 5?

- 1) 5 - single digit = 1
  - 2) 14, 41, 23, 32 = 4
  - 3) 111 (any permutation 3! = 6)
  - 4) 2 + 3 ones: 2111, 1211, 1121, 1112 = 4
  - 5) 11111 = 1
- 1 + 4 + 6 + 4 + 1 = 16

170. In  $\triangle ABC$ , shown here, the measure of  $\angle BCA$  is  $90^\circ$ ,  $AC = 12$  units and  $BC = 9$  units. If  $D$  is a point on hypotenuse  $AB$ , such that  $AD = 5$  units, what is the length of segment  $CD$ ? Express your answer in simplest radical form.



$$\begin{array}{l}
 9^2 + 12^2 = (AB)^2 \\
 81 + 144 = AB^2 \\
 225 = AB^2 \\
 15 = AB \\
 10 = BD \\
 \frac{10}{15} = \frac{2}{3}
 \end{array}$$

Draw  $\overline{DE} \parallel \overline{AC}$   
 $\triangle BED \sim \triangle BCA$   
 sides of  $\triangle BED = \frac{2}{3}$  of corresponding sides of  $\triangle BCA$

$$\text{so } \overline{ED} = \frac{2}{3}(12) = 8 \quad \text{and } \overline{BE} = \frac{2}{3} \text{ of } 9 = 6$$

$$\begin{array}{l}
 \text{so } \triangle EDC \text{ is rt } \triangle \text{ and} \\
 CD = \sqrt{8^2 + 6^2} \\
 = \sqrt{64 + 36} \\
 = \sqrt{100} \\
 = 10
 \end{array}$$